John Forester, M.S., P.E.

7585 Church St., Lemon Grove, CA 91945-2306
619-644-5481   forester@johnforester.com

Wednesday, May 28, 2008

How To Find Your Position At Sea: Very Elementary Celestial Navigation

1 Introduction: How to Make this Presentation

1.1 The Meaning and Use of Celestial Navigation

This presentation is intended to show the interested non-sailor how the sailor found his position at sea before the use of radio, loran, GPS, and other modern aids. The sailor navigated by either the sun or the stars, using the sextant to measure the altitude of the celestial body in the sky and the chronometer to measure the time of that observation, and calculating his position from these data and the navigational astronomical tables that he carried with him. These methods were the prime means of navigation from the introduction of the practical chronometer, just before Nelson's time, to the advent of electronic communications during World War II. These methods are still useful for the sailor deprived of electronics. Indeed, the author of this pamphlet once had the task of writing the specifications for the sextant-like instrument that was intended to guide the Apollo astronauts back to landing if all of their electronics failed.

1.2 How to Give this Presentation

1.2.1 Location

Part of the presentation will be made indoors or below decks, part outdoors or on deck. The outdoor or on-deck location needs to have a fairly level horizon below the sun (or a bright star) at the time of presentation. If the horizon is the sea, then the results may be reasonably accurate, but if the horizon is merely a distant hilltop or roofline of buildings, the students will still have the experience of pulling the sun (or star) down to that horizon with the sextant and measuring its altitude above that horizon, even though the results cannot provide an accurate position.

1.2.2 Equipment

The supporting instution needs to supply student sextants, globe, navigation tables. The other items are supplied by docent and students.

1: Sextants: One plastic student sextant for each three students. Before using any sextant, check out the alignment of the upper mirror against a fairly-level horizon and a fairly-vertical mast or similar, and align the mirror according to the instructions.

2: Watches: Each group of participants practicing with one sextant needs also to have one watch (preferably digital; probably there will be enough in the group).

3: Cards & Pencils: Each group of participants needs a three by five card to write upon, and a pen or pencil (the latter two items supplied by the docent).

4: Star map, Ephemeris, or Navigation Tables: The docent should also have a star map or ephemeris and navigation tables (doesn't matter what year).

5: Globe of the earth, on standard mount with axis 23.5° from vertical.

6: One to two feet of twine. This serves as a compass to indicate circles of position on the globe, without damaging the globe.

7: Something to indicate the Sun. A table lamp with bare 40W bulb, as high as the center of the globe, would be ideal.

1.2.3 Presentation Sequence

The first part of the presentation requires a level floor or deck, and is done indoors.

1.2.3.1 Sun Principles

Rotation Directions. The surface of the Earth moves always eastwards, counterclockwise when looking from the north pole. The Earth circles the Sun in the same counterclockwise direction when viewed from the north pole. (Both rotate in the same direction because both rotations are derived from the rotation of the initial cloud of gas that finally condensed into the solar system.)

Set the Sun in the center of the room or compartment and the Earth at a convenient distance from it, with the axis at midwinter day. Demonstrate the rotation of the Earth on its axis, pointing out latitude of the sun (southern tropic line) and the longitude of noon. Then move the Earth around the Sun, keeping the Earth's axis aligned as before, so that the Sun's latitude goes to 0 at the spring equinox, to the northern tropic line at midsummer, back to 0 at the fall equinox, and back to the southern tropic line at midwinter again. With a table of the Sun's noon latitude for each day of the year, and a sextant to measure how far north or south we are from the Sun's noon latitude we know our latitude on the Earth.

Now say that we have accurate clocks, chronometers, that rotate once (twice, but we use the numbers twice per day) each time the Earth rotates. If that clock is set so that it reads noon when it is noon at the Greenwich longitude, and we can observe the time at which noon occurs where we are, then that time tells us how far around the Earth we are from Greenwich. The Earth rotates at 15 degrees per hour, so if the time of our noon is at 8 pm by Greenwich time, we are at 120° W longitude.

There are many details that must be accounted for, but that is the principle of navigating by the sun.

1.2.3.2 Star Principles

You have shown the Earth circling the Sun. Everything else to be seen around the Earth, be it the corner of the compartment or an actual distant star, represents the stars which are so far away that the motion of the Earth around the Sun makes no perceptible changes. Relative to the stars, the Earth is circling the Sun as if fixed in place.

Point to an upper corner of the compartment, saying that it represents a star, and then to the spot on the Earth where that star would be directly overhead. This is that star's Ground Point. Then rotate the Earth once, as for a day, showing that the GP traces out a circle of latitude. Always and forever (within reasonable time) the GP for that star will be somewhere on that latitude circle.

To fix the GP for any object we need both latitude and longitude. That means that we must have a fixed reference direction in the Universe, like the Greenwich meridian on Earth. Like the Greenwich meridian, this was arbitrarily set by man. Move the globe of the Earth around the Sun until it reaches the vernal equinox. The zero-angle plane is that defined by the line of the axis of the Earth and the point that is the center of the sun. Since the movement of the Earth around the Sun is minuscule compared to the size of the Universe, the direction fixed by this plane is fixed in the Universe. We make our measurements for stars similar to longitude as angles around the Earth's axis from this reference plane. The direction of this Zero plane is called The First Point in Aries (for ancient reasons).

The position of the star is given by two numbers. the first is the degree of its latitude line, called its Declination. The second is how far around the star is from the zero plane, like its longitude, but called the Hour Angle because it represents the time difference as the Earth rotates from when the zero plane is directly overhead to when the star's plane is overhead.

With a sextant and a chronometer, we could fix our position just as with the sun, and that is what astronomers do on land, but that is impractical at sea because you can't see the horizon at night from which to measure altitudes.

Therefore we do something a bit different. We use three items of information: Declination of the star, Hour Angle of the star, and the exact time. The exact time tells us the angle of the Earth relative to the Zero plane. Then we know the latitude and longitude of the star's Ground Point at that moment.

In this calculation, time is a little complicated, because our clocks run on sun time while the Earth rotates on star time. As the Earth moves around the Sun, the line between Earth and Sun rotates accordingly, once per year. This line is the line for noon, so we have to know not only our time relative to noon, but also the direction of noon, the

Hour Angle of the Sun, for that day and time. Our chronometer tells us the time relative to the nearest noons. The navigation tables give us the Hour Angle for noon at that moment, the angle between the Sun and the zero plane. So we add or subtract, as appropriate, to combine the Hour Angles of the sun and the star to get the longitude of the GP of the star.

1.2.3.3 Circle of Position

At dawn or dusk, when we can see both stars and horizon, the sextant enables us to measure how far, to the fraction of a degree, we are from the GP of the star. If that measurement is 20 degrees, then we are at some point on the circle that is 20 degrees in radius centered on the GP of the star.

Indicate some convenient point on the globe as the calculated GP of the star. Say that the sextant measurement showed the star as 20° from the zenith. Take the length of twine and hold it at two places apart by the length equal to 20° of latitude on the globe that you have. Hold one end of that length at the GP, and indicate the circle formed at 20° from that GP. Our position is somewhere on that circle. With several circles of position, taken at nearly the same time (so the ship can't have moved much), the intersection of the circles tells us where we are. The radius of each circle can be also stated in nautical miles, since each degree of arc equals 60 nautical miles (each minute of arc equals 1 nautical mile).

1.2.3.4 Use of Sextant and Chronometer

The location for the presentation should be easily accessible to a position from which several groups of participants can observe the sun (or the star) and the horizon in that direction. It will not be practical to make a proper set of noon observations, so don't even try.

When the presentation gets to the section on using the sextant, move the group to that location and let them practice making observations. Divide the class into groups of three students. Each student in turn uses the sextant to pull the sun (or star) down to the horizon, and later reads the altitude scale, while one of the others tells the time and another writes down the time and the altitude.

2 How Do We Get Where We Want To Go?

When driving a car, you have a map (either in your head, for local places, or on paper, for distant ones), and you follow it and the road signs, according to a list of instructions, either in your memory or written down. If you get lost, you drive until you reach an intersection of two roads, read the names from the signs, and the map tells you where that intersection is relative to your destination. Very few motorists now carry a compass, which would tell you whether to turn right or left when you find a particular road.

There are no road signs at sea, but the navigator has instruments by which he learns which east-west line he is on, and by which he learns which north-south line he is on. The point where those two lines intersect is where he is, just as if he found the intersection of an east-west road and a north-south road.

The east-west line is a line of latitude, and is specified as so many degrees north (or south) of the equator. The north-south line is a line of longitude, and is specified as so many degrees east (or west) of Greenwich, near London, England. Greenwich was chosen because it was the location of the astronomical observatory at which the necessary observations of the stars and sun were made when England was the pre-eminent sea power.

Since the navigator's destination is shown on the marine charts at its particular latitude and longitude, if he knows his own latitude and longitude he can calculate the course to steer to reach it, and the compass will tell him which way to point the ship to steer that course. As he sails along, from day to day, he makes new observations and calculations, so that whatever errors of position are forced on his ship by wind, wave, and ocean current, each day he can calculate the proper course to steer to reach his destination.

Since these navigational methods are based on the positions of the sun and the stars in the sky, this type of navigation is called celestial navigation.

3 Basic Celestial Navigation Principles

Celestial navigation is based on a few simple principles.

3.1 The Sun Principles

The easiest way to fix our position is to measure how close the sun gets to being directly overhead at noon of that day. The angle between sun and the zenith is the zenith angle. We also need to measure the time of that noon relative to Greenwich time (chronometer time).

While the earth rotates on its own axis, it also rotates around the sun, once per year. However, the axis of the earth's own rotation is not parallel to the axis of its rotation around the sun. These two are tilted 23.5 degrees apart. This makes the seasons on earth, because at midsummer day the sun appears to rise to a maximum above the earth's horizon 47 degrees (twice 23.5 degrees) higher than its maximum height at midwinter day. In other words, the spot where the sun is directly overhead on northern midwinter day traces a line of latitude 23.5 degrees south of the equator, while on northern midsummer day that line is 23.5 degrees north of the equator. This seasonal progress is calculated, so that the navigational tables give the sun's latitude line for each day of the year. (The lines vary a little because of leap year and other matters, so the tables are slightly different for each year.)

Presuming that we know the date, then, by measuring the minimum zenith angle of the sun on that day from where we are, and adding that to sun's latitude line for this date, tells us our own latitude. This was the earliest form of celestial navigation.

Presuming that we also know Greenwich time, if we measure the Greenwich time at which the sun is highest at our location, the difference between Greenwich noon and local noon, calculated at 15 degrees per hour, the speed of rotation of the earth, tells us our longitude. This type of navigation could not be developed until we had clocks, called chronometers, accurate enough to run at constant speed for the duration of a voyage, despite the motion of the ship and the changes in temperature, humidity, and barometric pressure.

With these two measurements, we have a noon fix, we know where we are right then. We have our latitude and our longitude.

There are many complications that have to be accounted for by much arithmetic, which we don't go into here, but that is the principle.

3.1.1 Data for the Teaching Location

Determine the lattitude and longitude of the teaching location (most computer mapping programs provide this readily). Since this presentation was originally for the San Diego Maritime Museum, the following data are used in the example. Substitute the data for your location.

The position of the San Diego Maritime Museum is N 32° 43.135', W 117° 2.084'. Consider it N 33°, W 117° for this discussion. Consider the tilt of the Earth's axis to be 23.5° exactly.

On midsummer day, the sun will be 33° - 23.5° = 9.5° away from directly overhead, the zenith. Therefore its maximum altitude above the horizon on midsummer day (the maximum for any day) will be 90° - 9.5° = 80.5°. On midwinter day the sun will be 33° + 23.5° = 56.5° away from zenith, and therefore will have a maximum altitude (the lowest for any noon in the year) of 90° - 56.5° = 33.5°.

If the longitude of the Maritime Museum were W 120° exactly, then our noon would be exactly 8 hours behind the Greenwich noon. That is because 120 degrees is exactly 1/3 of the whole circle and 8 hours is exactly 1/3 of a day. At W 117°, we are 24 hours * 117 / 360 = 7.800 hours = 7 hours 48 minutes behind Greenwich. (7h 48m 42s for W 117° 2.084')

3.1.2 The Equation of Time: Days Aren't All the Same Length

The orbit (path) of the earth around the sun is not a circle, but an ellipse. Because of this shape of its orbit, the earth does not move in equal angles in equal time at all points of its orbit. As a result, for part of the year, noon for the sun is up to a few minutes earlier than shown by a clock, or by star time, while for the rest of the year it is up to a few minutes later than shown by a clock or by star time. Since all the celestial calculations are made according to star time, which goes constantly at the same rate, we have made tables that show the difference between clock noon and sun noon for each date of the year. These don't change very much, can be considered constant for the year. When observing local noon, you must then correct by this Equation of Time, as this table is known.

3.1.3 Not All Years Are The Same Length

You know that every four years, except for some specified occasions, we add a day to the year to keep the calendar matching the seasons. This requires regular recalculation of the navigational tables, so you must use the tables appropriate for the current year.

Also, every year we have two years of different lengths. Because the earth rotates on its own axis in the same direction that it rotates about the sun, in each year there are one more apparent rotation of the stars than there are apparent rotations of the sun. So the sidereal year (the star year) has one more "day" than does the sun year.

You don’t have to worry about that, but astronomical clocks run on star time, making each day about 3 min 56 secs shorter than a calendar day.

3.2 The Star Principles

Measuring the altitude of several stars and the times of those observations also enables us to fix our position on the earth. Some stars that are bright, easily identified, and well scattered around the celestial globe have been classified as navigation stars. Their positions are shown in the navigation tables to serve as the basis of calculation. Getting a star fix is more difficult than getting a sun fix, but you might get a star fix when you can’t get a sun fix.

1: The earth exists in a universe of extremely distant fixed stars. Because these stars are so far away, the apparent motion of these (they do move) is so small that it can be disregarded. For the same reason, the movement of the earth around the sun doesn’t affect their apparent position as seen from the earth. For our purposes, we can consider the stars as points of light fixed on the surface of a spherical universe with the earth as its center. This spherical universe has its latitude and longitude lines, somewhat like the earth has, although they are called declination and hour angle instead of latitude and longitude.[1]

2: The earth spins on its axis once a day, from west to east, always at the same speed. This speed is 360 degrees in 24 hours, or 15 degrees per hour. The speed of a point on the earth at typical middle latitudes is about 1 mile in 4 seconds, or 1/4 mile per second.

3: Because the earth spins on its axis, the universe of stars, as seen from the earth, appears to rotate once a day in the opposite direction, from east to west. This allows us to specify the poles and the equator for the supposedly rotating, supposedly spherical universe. The celestial equator is in the same plane as the earth’s equator. We can also fix a direction in that universe from which we can measure the hour angle (“longitude”) of stars. This direction is specified as the direction between the sun and the earth at the moment that spring arrives, the Vernal Equinox, when the sun crosses the earth’s equator from south to north. As a navigator, you don’t need to know what this direction is, because only the difference between two measurements is used.

4: Because the earth spins on its axis, the spot on the earth where a specific star is directly overhead (that is, the spot on the surface of the earth which is directly between the center of the earth and the distant star) traces out a line of latitude as the earth turns once a day. The number of degrees which this line is north or south of the equator is known as the declination of the star and is listed in the navigation tables.

5: The east-west direction of the star, comparable to longitude on earth, is known as the star’s Right Ascension or Hour Angle. It is defined as the angle between the plane of longitude fixed by the Vernal Equinox and the plane of longitude of the star. It may be expressed either in degrees or in time. This angle is listed in the navigation tables.

6: The actual longitudinal angle that we use is the difference between the Hour Angle of the sun at noon on that day, which is available from the navigation tables, and the Hour Angle of the star, which is also available from the same tables. That is why we don’t need to know the precise direction of the zero Hour Angle.

4 Finding Position At Noon

For finding the position at noon, you use the sun principles to make two measurements, the maximum altitude of the sun and the time at which it reaches that altitude. Those two measurements will give you a fix, two intersecting lines for which you are at the center.

4.1 What You Need To Know

You need to know two things in advance:

1: The maximum height of the sun, in degrees above the equator, on this day

2: The time according to the clock at Greenwich, near London.

The maximum height of the sun for this day is given in a table of navigational data. A simple table just lists the days of the year and the maximum angle for each day. However, this varies a bit from year to year and for accurate work you need to use the published tables for the current year.

The time according to the clock at Greenwich

1. Star positions given as latitude and longitude refer to a different system based on the plane of the earth’s orbit around the sun instead of the earth’s rotation about its own axis.

is given by the chronometer, a very accurate clock that is carried aboard ship.

4.2 What You Need To Measure

You need to measure two things:

1: The maximum altitude of the sun above the horizon on this day, which is the noon altitude. You measure this altitude to get the zenith distance, which is simply 90° - observed altitude.

2: The time of this local noon.

The altitude of the sun above the horizon is measured with a sextant (meaning the sixth part of a circle). The time of noon is measured by using both the sextant and the chronometer.

4.3 Using The Sextant

This discussion assumes that you are using a simple student sextant that is properly aligned.

If the weather is reasonable when you are at sea, you will be able to see the horizon line directly under the sun. If you are making a sun observation, you must have the dark filter in place between the top mirror and the horizon mirror. Without that, you will blind yourself by looking at the sun (much more dangerous through a telescopic sextant). Stand facing directly towards the sun with the sextant held vertically in your right hand. Look through the eyepiece toward the horizon, and move the sextant in small circles until you see an image of the sun on the horizon mirror. This will require both changing the angle of the sextant body and moving the arm that holds the upper (sun) mirror. Once you see the image of the sun (it will look like a red circle through the dark filter), manipulate the angle of the sextant as a whole, and the angle of the arm, so that you can pull the image of the sun down to where, on the horizon mirror, it is just adjacent to the line of the horizon. Holding the sextant exactly vertical and steady, by moving its arm, bring the sun's image so that it is centered on the line of the horizon. To determine whether you are holding the sextant exactly vertical, rock it slightly. You will see the image of the sun rise a little on each side of a low point. When the sun is at the low point, you are holding the sextant exactly vertical. Adjust the position of the sextant's arm to get the center of the sun on the line of the horizon. That is the reading that you want. Read the point where the zero mark of the arm crosses the scale of degrees at the bottom of the sextant's body.

The zero mark will not be exactly aligned to any particular degree mark, but will show some full degree plus a fraction of a degree. The specific fraction can be read from the vernier scale that is opposite the main scale. The vernier scale will be marked up to 60 minutes (1degree has 60 minutes). Look along the vernier scale to find the mark that best aligns with another degree mark. The designation of this mark is the fraction of a degree to be added to the last whole degree indicated on the main scale. Read and record the full reading in degrees and minutes.

You have now "shot the sun" once. It takes practice to do it well and rapidly. A proper navigational sextant works the same way, but has more features. It will have a low-power telescope rather than a sighting hole, a greater selection of optical filters for different conditions, a screw-operated vernier system for greater ease and accuracy, scales reading to fractions of minutes rather than fractions of degrees. However, it operates according to the same principles and with the same methods.

4.3.1 Correcting for Errors

Your sextant might have a built-in error, although one tries to adjust such errors out. That error must be subtracted from the reading. The horizon line that you see is not quite level with your eye (that is, tangent to the sphere of the ocean at your location), because of two factors. The first is that your eye is above the water level. That correction is available from tables, provided that you know your own height above the water. The second is that the path of light is curved by the changes in density of the air along the path. Correcting that error is beyond the scope of this discussion.

It is also often better to observe the edges of the sun (the bottom limb generally) and position one of them on the horizon. Setting the sun so it sits just on the horizon line is considered most accurate. Then the correction for the radius of the sun must be applied. That is given at the front of the tables.

The corrections for all errors must be applied to the sextant reading before using the reading for final fix.

4.4 Using The Chronometer

Of course, the chronometer must have been operating properly since it was last set to Greenwich time. If it hasn't been operating properly, it is useless unless it is checked for proper rate of error and is again set to Greenwich time. In the old days, this had to be done ashore at a proper chronometer laboratory. Nowadays, with radio reception, an accurate quartz watch can be set frequently and its rate of error ascertained. Therefore, for many purposes, the quartz watch whose rate is known, recently set by radio, is adequate.

Remember, at typical latitudes, the surface of the earth is spinning eastwards at 1/4 mile per second. Therefore, each four seconds of error in the time amounts to a mile in longitude. There are enough sources of error; don't add chronometer error to those if you can help it.

The chronometer is used only with the sextant; it is useless for navigation without the sextant. Therefore, you have to use them both together. The most accurate way to use the pair is to have one person for each. The person using the sextant gets his fix on the sun (or star, at dusk or dawn) and says "Mark!" The person watching the sextant records the time shown at that word. Then the sextant's reading is taken and is recorded against the time reading.

If only one person is doing the navigation, he must work steadily so that he takes the same amount of time to go from taking the fix to observing the chronometer. Then he records the observed time, then the sextant reading, and later corrects the time for the duration between taking the fix and seeing the face of the chronometer. A stopwatch makes this easier. He must also make the corrections required by the difference between chronometer time and Greenwich time, based on the seconds lost or gained per day and the number of days since last setting. Only then can use the calculated time to work out his fix.

4.5 Getting the Noon Position

4.5.1 Determining Noon Sun Altitude

In most vessels, you won't have an enormous difference in noon sun altitude from day to day. Therefore, you know just about when it will be. You also know just about when local noon will be, because it won't be much different from 24 hours after yesterday's noon.

Therefore, about half an hour before anticipated noon, start shooting the sun and recording times and altitudes. The altitude will be increasing between readings. Then the rate of increase will slow down and finally stop. Here is where the screw operated verniers are a great assistance, but with care you can do pretty well even with a student sextant. Record the maximum altitude that you observe, and its time if possible.

4.5.2 Determining Local Noon Time

You don't use this time as the time of local noon, because it is a fuzzy time and there is a much more accurate way. Look down your list of times and altitudes to find one near the start that you think is reliable and whose angle is easy to set accurately. Set your sextant to the same reading. Keeping the sextant set at that angle, observe the sun until it descends to the horizon line. The moment that it does so, observe the time and record it.

The time of noon is halfway between the times of the two observations of equal altitude. You can make several of these observations and average the times to determine local noon.

4.5.3 Calculating the Latitude

You know from the compass (and possibly your sextant reading if the sun is almost directly overhead) whether you are north of the sun or are south of the sun. The tables give the latitude of the sun at noon for that day. If you were on that latitude, the sun would be directly overhead at noon. However, you are probably not on that line. Subtracting your sun's measured altitude above the horizon from 90 degrees gives you the angle that you are from the sun's latitude line. Then, adding that difference (using proper signs for north and south) to the sun's latitude line gives you your latitude.

4.5.4 Calculating the Longitude

The difference between your time of local noon and Greenwich noon (calculated by the corrected chronometer time) indicates your longitude. If your noon is earlier than Greenwich's noon, then you are east of Greenwich and your longitude will be in degrees east. If your noon is later than Greenwich's noon, you are west of Greenwich and your longitude will be in degrees west. The number of degrees will be the time difference in hours multiplied by 15 because the earth rotates at 15 degrees per hour. With a calculator, It is easiest to convert the time difference to seconds, and then multiply by 15 (degrees per hour) and divide by 3600 (seconds per hour). This gives you the number of degrees east or west of Greenwich.

4.5.5 The Final Noon Position

The noon sun altitude gives you latitude, and the noon sun time gives you longitude. These two numbers fix your position on the chart and at sea.

5 Finding Position at Dawn or Evening

If you couldn't take a noon sight of the sun (couldn't see the sun, or couldn't see the horizon), you may be able to take dawn or evening sights of several stars. It must be dark enough to see the stars, yet light enough to see the horizon. Sights of two stars theoretically will give you two intersecting lines on the chart to fix your position, but sights of three stars are much better. Those three stars will give you three lines of position that should meet to make a very small triangle on the chart.

However, computing the line of position that is determined by the measurement of the star's altitude and the time of the observation is much more complicated than making a sun sight. We will just consider the principle without describing the details. To do the work, you need to know spherical trigonometry, the mathematics of triangles that are drawn on the sphere of the earth, and that is just too complicated for this discussion.

5.1 The Principle of the Star Sight

Remember that the stars are fixed in position, and the earth rotates on its axis once a day. Of course, you can see only half of the stars on any one night, those visible from the dark side of the earth. However, six months later, when the earth has made half a circle around the sun, the other half of the stars are visible. Therefore, in the course of a year, all the stars have been visible.

5.1.1 Why We Must Use More Mathematics To Calculate From A Star Sight

At all times, some point on the earth's surface is directly below a particular star. This is the place where a line going directly from the center of the earth to the star would come through the surface of the earth, and it is the place on the earth where that star is directly overhead, has an altitude of 90 degrees no matter from what direction you measure it. That place is named the star's Ground Point, or GP.

Because the earth's axis is fixed in the universe of stars, doesn't change direction relative to the stars as the earth rotates, the GP for a particular star traces out a circle of latitude on the earth. That latitude is the same number of degrees as the Declination of the star, one of the two numbers that specify the position of that star on the sphere of stars. When shooting the sun, we used the altitude of the sun at noon, which gave us our latitude, and the time of noon to find our longitude. In theory, we could do just that from a star sight, but that doesn't work because when that star is highest in the sky the night has made the horizon invisible. We have to measure the altitude of the star at the time that both it and the horizon are visible. That's why we have to use a different technique with much more mathematics when computing from a star sight.

5.1.2 The Circle of Position For A Star Sight

The nautical mile (6080 feet) is defined as the distance measured on the sphere of the earth (at the Equator, or on any other Great Circle path) for an angular change of one minute, the 60th part of a degree. Now here's the mathematical trick we use. If you were at the star's Ground Point, its altitude would be 90 degrees, because that's the definition of the GP. But suppose that you weren't at the GP, but from where you were you measured its altitude as 80 degrees. That is, 10 degrees less than the 90 degrees at the GP. That means that you are somewhere on the circle where the surface of the earth has curved 10 degrees from its surface at the GP. Since 10 degrees is 600 minutes of arc, you are someplace that is 600 nautical miles from the GP, because a nautical mile is defined as the distance for each minute of arc on the earth's surface. Of course, you don't know the precise direction (have only a hazy notion of) from you to the GP, but certainly you are somewhere on the circle with radius 600 miles that is centered on the GP for that star at that moment in time.

If you can calculate the position of the star's GP at the moment that you measured its altitude, then you know on which circle you must be, and you can draw it on the chart. If you can do that for three circles, each computed for a different star whose altitude and time of observation were taken no more than a few minutes from the other two (in other words, from the same position at sea), then you must be at the only point where these three circles intersect.

5.1.3 Calculating the Ground Point for a Star and a Time

The latitude of the star's Ground Point at the time you measured the star's altitude is easy. That latitude never changes. It is the latitude with the same number of degrees as the star's Declination, which is given in the navigation tables.

Now you need to find the longitude of the GP. The navigation tables give you the Hour Angle for the star. That is, the angle between the direction of the star and the direction of the Vernal Equinox that is the zero longitude for the universe. However, that can't be the longitude on earth because the earth is rotating in two ways, on its axis and also around the sun. Greenwich noon is defined, each day, as when the Greenwich longitude zero passes directly under the sun. However, as the earth circles the sun once in each year, the direction between the sun and the earth rotates a full circle over the course of the year. Because the earth actually follows an elliptical path instead of a circle, this motion is not quite constant. However, it is tabulated in the navigation tables. By using those tables and your exact time of observation, with a bit of arithmetic you can determine the direction in which the Greenwich longitude zero pointed when you made your observation. This is the Hour Angle for the Greenwich longitude zero at that date and time of day. The difference between the Hour Angle for Greenwich and the Hour Angle for the star gives you the longitude on the earth.

So now you have the location of the star's Ground Point, its latitude and longitude. You also have the radius of the circle around that point on which you must be (from the observed altitude), and, in theory, you can draw that circle on the chart.

By doing the same thing for three different stars, you get three circles. Three circles that intersect each other can have six points of intersection, but there can be only one point at which all three intersect. You must be at that point on the chart and at sea.

5.2 The Lines of Position

However, it is inconvenient to try to draw those actual circles on the chart, because the centers of the circles are too far away off the chart, way far away from the ship probably, on a chart of useful scale. If you used a chart with scale small enough so you could draw the circles, then you would not be able to read your position with any useful accuracy. Therefore, you draw what are called lines of position around an assumed position. This is where the spherical trigonometry comes in, making many details that are too complicated to discuss herein, so only the principle will be described here.

You know pretty well where your ship is. So, you start by assuming that it is where you think it is. Now, the bit of any circle of position that is near your ship will be substantially straight, because these circles are so large. For each star sight, you compute the direction and distance between your assumed ship's position and the GP of the star. You then subtract your assumed distance from the GP from the distance you know you are from that GP. (That's what you measured with the sextant.) You then draw the straight line that represents the small portion of the circle of position that is near your assumed ship's position, at right angles to the direction to the GP and the proper distance nearer or further away from the assumed position.

After you have done that for three stars, the three lines of position should form a small triangle, and you should be very close to the center of that triangle.

If the triangle is too large and far distant from your assumed position, you can choose a new assumed position near the center of that triangle and work the trigonometry again, to see if you then get a small triangle. If that doesn't work, then you have made some error, perhaps in the initial observation. So if you have four sights instead of three, you can see which one to discard, the one that is at greatest deviation from the others.

5.3 Which Stars to Shoot?

To be useful for navigation, a star must be bright, easily recognizable, and in a useful part of the sky relative to the other navigational stars. The navigation tables list the names and positions (declination and hour angle) of the navigational stars. But you have to be able to tell which star is which, particularly in the evening when the stars are just starting to appear. If you have learned the constellations and the navigational stars in them, and if you have been paying attention to the stars at dusk and dawn for the time in which you have been sailing, then you should be able to identify the navigational stars that appear in your sky at evening before the horizon disappears. Not only will you know which star is which, but you will know the general direction in which it is to be found at dusk or dawn.

5.4 Shooting the Stars

The technique for making a sight on a star is much the same as for making a sight on the sun, except you cannot expect to do it for the “noon” of the star, the time at which it reaches its maximum altitude. Therefore, each sight, to be of any use at all, must combine the altitude of the star and the time of the observation. Remember, that each four seconds of error in time will make you about 1 mile wrong in your observation.

Before you start, you should have decided which three or four stars to shoot, spread around the horizon as much as possible and rather high in the sky. write their names on your card. As the stars become visible and the horizon fades, work as quickly as possible, preferably starting in the direction in which the horizon is first fading. Take the altitude of each star and the time of the observation, write them down adjacent to the name, then go to the next star.

To shoot the stars at dawn, you will have had time to easily identify the stars you want to shoot. Start shooting the stars as soon as the horizon becomes visible directly beneath each star.

6 Dawn and Noon Sun Sights

A common method for taking sights when at sea is to use sun sights at early morning, noon, and late afternoon. This more often allows the observations to be taken when the horizon is well-defined and the celestial target is easy to see. The noon sight is taken as described above. The morning or afternoon sights are taken and computed using the Ground Point and circles of position for the sun at the time of the observation. The noon sight gives a good latitude and a fair longitude. The morning and afternoon sights, where the sun is considerably to the east or west, give good longitudes and fair latitudes. Of course, the ship will have moved during the day, but you probably have a pretty good estimate of the direction and distance. You need to correct the lines of position calculated from each observation by the estimated distance and direction moved between observations. Then the intersection of the noon and morning, or noon and evening, lines of position gives your position. If all three intersect in a small triangle, then your work is probably accurate.